HASSAN UNIVERSITY

Department of Studies in Mathematics
HEMAGANGOTHRI, HASSAN

Regulations and Syllabus
Master of Mathematics (M.Sc.)
(Two-year semester scheme)

Under
Choice Based Credit System
(CBCS) and (FCBCS)

UNIVERSITY OF HASSAN
GUIDELINES AND REGULATIONS LEADING TO MATHEMATICS SCIENCE (TWO

YEARS-   SEMESTER SCHEME UNDER CBCS)

Programme Details

Name of the Department	: Department of Studies in Mathematics
Subject	: Mathematics
Faculty	: Science and Technology
Name of the Course	: Master of Mathematics (M.Sc.)
Duration of the Course	: 2 years- divided into 4 semesters

Programme Outcomes

At the end of the programme, the students will be able to:

1. Apply knowledge of Mathematics, in all the fields of learning including higher research and its extensions.

2. Innovate, invent and solve complex mathematical problems using the knowledge of pure and applied mathematics.

3. Explain the knowledge of contemporary issues in the field of Mathematics and applied sciences.

4. Adjust themselves completely to the demands of the growing field of Mathematics by life-long learning.

5. Effectively develop good writing and presenting skills for perusing research in Mathematics.

6. Crack lectureship and fellowship exams approved by UGC like CSIR – NET and SET.

7. Prepare with good motivation for research studies in Mathematics and related fields.

8. Develop required communication and computing skills to peruse a career in teaching field or software industry.

Programme Specific Outcomes

1. Understand the fundamental axioms in mathematics and capability of developing ideas based on them.

2. Inculcate mathematical reasoning.

3. Conduct research studies in mathematics and related fields.

4. Provide knowledge of a wide range of mathematical techniques and application of mathematical methods/tools in other scientific and engineering domains.

5. Acquire Provide advanced knowledge on topics in pure mathematics, empowering the students to pursue higher degrees at reputed academic institutions.

6. Get strong foundation on algebraic topology and representation theory which have strong links and application in theoretical physics, in particular string theory.

7. Acquire good understanding of number theory which can be used in modern online cryptographic technologies.

8. Equip in problem solving skills, thinking, creativity through assignments, project work.

9. Assist students in preparing (personal guidance, books) for competitive exams e.g. NET, GATE, etc.

Programme Pedagogy

The pedagogy of teaching-learning involves three components.
- Lectures with intellectual inputs form the first component. This method provides Receptive Instructions to students.
- The second component is the tutorials. This method provides Directive Instructions to students.
- The third major component is the practical orientation with skills and participatory learning works. This method involves Exploratory Instructions.

Programme Structure

Sl. No.	Stream Code	Course Code	Type of the paper	Title of the Paper	No.of Credits L:T:P	Total Credits
				I SEMESTER		
1	MSMAT13	18101	HC	Algebra I	3:1 :0	4
2	MSMAT13	18102	HC	Real Analysis I	3:1:0	4
3	MSMAT13	18103	HC	Real Analysis II	3:1:0	4
4	MSMAT13	18104	HC	Complex Analysis I	3:1:0	4
5	MSMAT13	18105	SC	Linear Algebra	3:1:0	4
6	MSMAT13	18106	SC	Combinatorics and Graph Theory	3:1:0	4
				II SEMESTER		
7	MSMAT13	18121	HC	Algebra II	3:1:0	4
8	MSMAT13	18122	HC	Real Analysis III	3:1:0	4
9	MSMAT13	18123	HC	Complex Analysis II	3:1:0	4
10	MSMAT13	18124	SC	Ordinary and Partial Differential Equations	3:1:0	4
11	MSMAT13	18125	SC	Representation Theory of Finite Groups	3:1:0	4
12	MSMAT13	18126	OE	Discrete Mathematics	3:1:0	4
				III SEMESTER		
13	MSMAT13	18141	HC	Elements of Functional Analysis	3:1:0	4
14	MSMAT13	18142	HC	Topology I (Topology - FCBCS)	3:1:0	4
15	MSMAT13	18144	SC	Theory of Numbers	3:1:0	4
16	MSMAT13	18107	SC	Graph Theory	3:1:0	4
17	MSMAT13		SC	Theory of Modular Forms	3:1:0	4
18	MSMAT13	18143	SC	Commutative Algebra	3:1:0	4
19	MSMAT13	18145	SC	Algebraic Number Theory	3:1:0	4
20	MSMAT13	18146	SC	Galois Theory	3:1:0	4
21	MSMAT13	18147	OE	Differential equations and its applications	3:1:0	4
				IV SEMESTER		
22	MSMAT13	18161	HC	Measure and Integration	3:1:0	4
23	MSMAT13	18162	HC	Topology II (Advanced Topology - FCBCS)	3:1:0	4
24	MSMAT13	18163	SC	Differential Geometry	3:1:0	4
25	MSMAT13	18164	SC	Advanced Graph Theory	3:1:0	4
26	MSMAT13	18165	SC	Theory of Partitions	3:1:0	4
27	MSMAT13	18168	SC	Hypergeometric Functions and q - Series	3:1:P	4
28	MSMAT13	18166	SC	Advanced Functional Analysis	3:1:0	4
29	MSMAT13	18167	SC	Algorithms and computations	3:1:0	4

COURSE-I: ALGEBRA I

Course Outcome:

At the end of the course, the students will be able to:

- Apply the knowledge of Algebra to attain a good mathematical maturity and enables to build Mathematical thinking and skill.

- Utilize the class equation and Sylow theorems to solve different related problems.

- Identify and analyze different types of algebraic structures such as Solvable groups,

- Simple groups, Alternate groups to understand and use the fundamental results in Algebra.

- Design, analyze and implement the concepts of homomorphism and isomorphism between groups for solving different types of problems, for example, Isomorphism theorems, quotient groups, conjugacy etc.

- Create, select and apply appropriate algebraic structures such as finitely generated abelian groups, Ideals, Fields to explore the existing results.

- Identify the challenging problems in modern mathematics and find their appropriate solutions.

Course Content

UNIT- I

Number theory - Congruences, residue classes, theorems of Fermat, Euler and Wilson, linear congruences, elementary arithmetical functions, primitive roots, quadratic residues and the law of quadratic reciprocity.

UNIT -II

Groups - Lagrange's Theorem, homomorphism and isomorphism, normal subgroups and factor groups.

UNIT- III

The fundamental theorem of homomorphism, two laws of isomorphism.

UNIT- IV

Permutation groups and Cayley's theorem, Sylow's theorems.

Books for Reference:

1. D. M. Burton – Elementary Number Theory, Tata McGraw-Hill, New Delhi, 6thEd.,

2. I. Niven, H. S. Zuckerman and H. L. Montgomery – An Introduction to the Theoryof Numbers, New York, John Wiley and Sons, Inc., 2004, 5thEd.,

3. G. A. Jones and J. M. Jones – Elementary Number Theory, Springer,1998.

4. Thomas W. Hungerford – Algebra, Springer International Edition, NewYork.

5. Michael Artin – Algebra, Prentice-Hall of India, NewDelhi.

6. J. A. Gallian – Contemporary Abstract Algebra, Narosa Publishing House, 4thEd.,

7. D. S. Dummit and R. M. Foote – Abstract Algebra, John Wiley and Sons,1999.

8. I. N. Herstein – Topics in Algebra, Vikas Publishing House, NewDelhi.

9. J. B. Fraleigh – A First course in Abstract Algebra,Addison-Wesley,

10. N. S. Gopalakrishnan – University Algebra, New Age International, 2ndE

COURSE-II: REAL ANALYSIS I

Course Outcome:

At the end of the course, the students will be able to:

- Describe the real line as a complete, ordered field.

- Identify challenging problems in real variable theory and find their appropriate solutions.

- Deal with axiomatic structure of metric spaces and generalize the concepts of sequences and series, and continuous functions in metric spaces.

- Use theory of Multiplications of series and infinite products in solving problems arising in different fields of science and engineering.

- Extend their knowledge of real variable theory for further exploration of the subject for going into research.

Course Content

UNIT- I

The extended real number system, the n-dimensional Euclidean space, the binomial inequality, the inequality of the arithmetic and geometric means, the inequality of the power means, Cauchy's, Holder's inequality and Minkowski's inequality.

UNIT- II

Numerical sequences, convergent sequences, Cauchy sequences, upper and lower limits.

UNIT -III

Series of real numbers series of non-negative terms, the number `e', tests of convergence.

UNIT -IV

Multiplications of series, re-arrangements. Double series, infinite products.

Books for Reference:

1. W. Rudin – Principles of Mathematical Analysis, International Student edition, McGraw Hill, 3rdEd.

2. T. M. Apostal – Mathematical Analysis, Addison Wesley, Narosa, New Delhi, 2ndEd.

3. R. R. Goldberg – Methods of real Analysis, Oxford and IBH, NewDelhi.

4. Torence Tao – Analysis I, Hindustan Book Agency, India,2006.

5. Torence Tao – Analysis II, Hindustan Book Agency, India, 2006.

6. Kenneth A. Ross – Elementary Analysis: The Theory of Calculus, Springer International Edition,2004.

COURSE-III: REAL ANALYSIS II

Course Outcome:

- Apply the knowledge of concepts of functions of real variables in order to study theoretical development of different mathematical concepts and their applications.
- Understand the nature of abstract mathematics and explore the concepts in further details.
- Determine the continuity, differentiability and integrability of functions defined on subsets of the real line.
- Utilize the concepts of derivative, MVTS for real - valued functions applied to different fields of science and engineering.
- Use theory of Riemann – Stieltjes integral in solving definite integrals arising in different fields of science and engineering.

Course Content

UNIT- I

Finite, countable and uncountable sets, the topology of the real line.

UNIT- II

Continuity, uniform continuity, properties of continuous functions, discontinuities, monotonic functions.

UNIT- III

Differentiability, mean value theorems, L' Hospital rule, Taylor's theorem, maxima and minima, Functions of bounded variation.

UNIT- IV

The Riemann-Stieltje's integral, criterion for integrability. Properties of the integral, classes of integrable functions. The integral as the limit of a sum. First and second mean value theorems. Integration and differentiation.

Books for Reference:

1. W. Rudin – Principles of Mathematical Analysis, International Student edition, McGraw-Hill, 3rdEd..

2. Torence Tao – Analysis I, Hindustan Book Agency, India,2006.

3. Torence Tao – Analysis II, Hindustan Book Agency, India, 2006.

4. T. M. Apostal – Mathematical Analysis, Addison Wesley, Narosa, 2ndEd.,

5. R. R. Goldberg – Methods of real Analysis, Oxford and IBH Publishing Company, NewDelhi.

6. Kenneth A. Ross – Elementary Analysis: The Theory of Calculus, Springer International Edition,2004.

COURSE-IV: COMPLEX ANALYSIS I

Course Outcome:

- Understand how complex numbers provide a satisfying extension of the real numbers.

- Solve real integrals by doing complex integration.

- Work on Taylor series of a complex variable illuminating the relationship between real function.

- Learn techniques of complex analysis that make practical problems easy (e.g. graphical rotation and scaling as an example of complex multiplication).

- Continue to develop proof techniques.

Course Content

UNIT- I

Algebra of complex numbers, geometric representation of complex numbers. Riemann sphere and Stereographic projection, Lines, Circles. Limits and Continuity.

UNIT- II

Analytic functions, Cauchy-Riemann equations, Harmonic functions, Polynomials and

Rational functions. Elementary theory of power series - sequences, series, uniform

convergence of power series, Abel's limit theorem, The elementary functions.

UNIT -III

Topology of the complex plane. Linear fractional transformations, Cross-ratio,
Symmetry, Elementary conformal mappings. Complex integration – Line integrals,
Rectifiable arcs.

UNIT- IV

Cauchy's theorem for a rectangle. Cauchy's theorem in a Circular disk,
Cauchy's integral formula. Local properties of analytic functions.

Books for Reference:

1. L. V. Ahlfors – Complex Analysis, McGraw-Hill, Kogakusha,1979.

2. J. B. Conway – Functions of one complex variable, Narosa, NewDelhi.

3. R. P. Boas – Invitation to Complex Analysis, The Random House,1987

4. B. C. Palka – An Introduction to Complex Function Theory, Springer,1991.

5. S. Ponnusamy – Foundations of Complex Analysis, Narosa,1995.

<center>**SOFT CORE**</center>

COURSE-V: LINEAR ALGEBRA

Course Outcome:

- Understand the concepts of Liner independence, bases and Dual spaces.
- Discuss Algebra of Linear Transformations and Characteristics roots.
- Study canonical forms and Nilpotent transformations.
- Analyze rational canonical forms and Determinants.
- Understand the Hermitian, Unitary and Normal Transformations.

Course Content

UNIT- I

Vector Spaces, Subspaces, Linear Combinations and Systems of Linear
Equations, Linear Dependence and Linear Independence, Bases and
Dimension, Maximal Linearly Independent Subsets;

Linear Transformations, Null Spaces, and Ranges, The Matrix Representation of a Linear Transformation, Composition of Linear Transformations and Matrix Multiplication, Invertibility and Isomorphisms, The Change of Coordinate Matrix, The Dual Space;

Elementary Matrix Operations and Elementary Matrices, The Rank of a Matrix and Matrix Inverses, Systems of Linear Equations.

UNIT -II

Properties of Determinants, Cofactor Expansions, Elementary Operations and Cramer's Rule; Eigenvalues and Eigenvectors, Diagonalizability, Invariant Subspaces and the Cayley-HamiltonTheorem; Inner Products and Norms, The Gram-Schmidt Orthogonalization Process and Orthogonal Complements.

UNIT- III

The Adjoint of a Linear Operator, Normal and Self-Adjoint Operators, Unitary and Orthogonal Operators and Their Matrices, Orthogonal Projections and the Spectral Theorem; Bilinear and Quadratic Forms;

UNIT- IV

The Diagonal form, The Triangular form; The Jordan Canonical Form; The Minimal Polynomial; The Rational Canonical Form.

Books for Reference:

1. S. Friedberg, A. Insel, and L. Spence - Linear Algebra, Fourth Edition, PHI,2009.

2. Jimmie Gilbert and Linda Gilbert – Linear Algebra and Matrix Theory, Academic Press, An imprint ofElsevier.

3. I. N. Herstein – Topics in Algebra, Vikas Publishing House, NewDelhi.

4. Hoffman and Kunze – Linear Algebra, Prentice-Hall of India, 1978, 2ndEd.,

5. P. R. Halmos – Finite Dimensional Vector Space, D. Van Nostrand,1958.

6. S. Kumeresan – Linear Algebra, A Geometric approach, Prentice Hall India,2000.

COURSE-VI: COMBINATORICS AND GRAPH THEORY

Course Outcome:

- Learn Basic concepts of Lattice theory and Boolean algebra and apply it to switching circuits.

- Develop the skill to apply permutations and combinations in practical problems.

- Understand the definitions namely, cut vertex, bridge, block and block graph.

- Study the properties of trees and connectivity.

Course Content

UNIT- I

Partially ordered sets, Lattices, Complete lattices, Distributive lattices, Complements,

Boolean Algebra, Boolean expressions, Application to switching circuits.

UNIT- II

Permutations and Combinations, Pigeon-hole principle, Principle of inclusion and exclusion.

UNIT- III

Graphs, Vertices of graphs, Walks and connectedness, Degrees, Operations on graphs, Blocks - Cutpoints, bridges Block graphs and Cutpoint graphs.

UNIT- IV

Trees - Elementary properties of trees, Center, Connectivity, Connectivity and line

connectivity, Menger's theorem, Partitions, Coverings, Coverings and

independence number.

Books for Reference:

1. C. L. Liu – Elements of Discrete Mathematics, McGraw-Hill, 1986.

2. Kenneth H. Rosen – Discrete Mathematics and its Applications, McGraw-Hill, 2002.

3. F. Harary – Graph Theory, Addition Wesley Reading Mass, 1969.

4. N. Deo – Graph Theory With Applications to Engineering and Computer Science, Prentice Hall of India, 1987.

5. K. R. Parthasarathy – Basic Graph Theory, Tata McGraw-Hill, New Delhi, 1994.

6. G. Chartand and L. Lesniak – Graphs and Diagraphs, wadsworth and Brooks, 2 nd Ed.,

7. Clark and D. A. Holton – A First Look at Graph Theory, Allied publishers.

8. D. B. West – Introduction to Graph Theory, Pearson Education Inc.,2001, 2nd Ed.,

9. J. A. Bondy and U. S. R. Murthy – Graph Theory with applications, Elsevier, 1976.

SECOND SEMESTER

HARD CORE

COURSE-I: ALGEBRA II

Course Outcome:

After completing this course, the student will be able to:

- Learn the fundamental theorems of algebraic structures.

- Explore the concepts of Polynomial rings, UFD, ED, PID, Field extensions, Einstein's irreducibility criterion, Galois extensions etc.

- Learn how to apply the concepts in real-life situations.

- Apply the knowledge of Algebra to attain a good mathematical maturity and enables to build mathematical thinking and reasoning.

- Identify and analyze different types of algebraic structures such as Algebraically closed fields, Splitting fields, Finite field extensions to understand and use the fundamental results in Algebra.

- Design, analyze and implement the concepts of Gauss Lemma, Einstein's irreducibility criterion, separable extensions etc.

- Create, select and apply appropriate algebraic structures such as Galois extensions, Automorphisms of groups and fixed fields, Fundamental theorem of Galois theory to understand and use the Fundamental theorem of Algebra.

- Identify the challenging problems in advanced Algebra to pursue further research.

Course Content
UNIT- I

Rings, Integral domains and Fields, Homomorphisms, Ideals and Quotient Rings, Prime and Maximal ideals.

UNIT- II

Euclidean and principal ideal rings, Polynomials, Zeros of a polynomial, Factorization, Irreducibility criterion.

UNIT- III

Adjunction of roots, algebraic and transcendental extensions, Finite fields.

UNIT- IV

Separable and inseparable extensions, Perfect and imperfect fields. Theorem on the primitive element.

Books for Reference:

1. Thomas W. Hungerford – Algebra, Springer International Edition, NewYork.

2. Michael Artin – Algebra, Prentice-Hall of India, NewDelhi.

3. Joseph A. Gallian – Contemporary Abstract Algebra, Narosa, 4thEd.,

4. D. S. Dummit and R. M. Foote – Abstract Algebra, John Wiley and Sons,1999, 2ndEd.,

5. I. N. Herstein – Topics in Algebra, Vikas Publishing House, NewDelhi.

6. J. B. Fraleigh – A First course in Abstract Algebra,Addison-Wesley,

7. N. S. Gopalakrishnan – University Algebra, New Age International, 2nded.,

COURSE-II: REAL ANALYSIS III

Course Outcome:

- Apply the Mean Value Theorem and the Fundamental Theorem of Calculus to
  problems in the context of real analysis.

- Write solutions to problems and proofs of theorems that meet

  rigorous standards based on content, organization and coherence,

  argument and support, and style and mechanics.

- Determine the improper Riemann integrals and prove a selection of theorems
  concerning improper integrals.

- Recognize the difference between point wise and uniform convergence of a sequence
  of functions.

- Illustrate the effect of uniform convergence on the limit function with respect
  to continuity, differentiability, and integrability.

- Recognize the difference between calculus of one variable and several variables.

Course Content

UNIT- I

> Sequences and series of functions, Uniform convergence, Uniform convergence
>
> and continuity, Uniform convergence and integration, Uniform convergence and
>
> differentiation.

UNIT- II

> Power series, The exponential and logarithmic functions, The trigonometric functions.
>
> Improper integrals and their convergence.

UNIT III

> Functions of several variables, partial derivatives, continuity and differentiability,
> the chain rule, Jacobians.

UNIT -IV

The Implicit function theorem, Taylor's theorem, the Maxima and Minima, Lagrange's multipliers.

Books for Reference:

1. W. Rudin – Principles of Mathematical Analysis, International Student edition, McGraw-Hill, 3rdEd.,

2. T.M. Apostal – Mathematical Analysis, Addison Wesley, Narosa, 2ndEd.,

3. R.R. Goldberg – Methods of Real Analysis, Oxford and IBH, NewDelhi.

4. D.V. Widder – Advanced Calculus, Prentice Hall of India, New Delhi, 2ndEd.,

5. Torence Tao – Analysis I, Hindustan Book Agency, India, 2006.

6. TorenceTao – Analysis II, Hindustan Book Agency, India, 2006.

7. Kenneth A. Ross – Elementary Analysis: The Theory of Calculus, Springer International Edition,2004.

COURSE-III: COMPLEX ANALYSIS II

Course Outcome:

- Apply the concept and consequences of harmonicity and of results on harmonic and entire functions including the fundamental theorem of algebra.
- Analyze sequences and series of analytic functions and types of convergence.
- Evaluate complex contour integrals directly and by the fundamental theorem, apply the Cauchy integral theorem in its various versions, the Cauchy integral formula.
- Represent functions as Taylor, power and Laurent series and classify singularities and poles.
- Find residues and evaluate complex integrals using the residue theorem.

Course Content

UNIT- I

The Calculus of Residues – The residue theorem, argument principle, Evaluation

of definite integrals.

UNIT- II

Harmonic functions – Definition and basic properties, mean value property, Poisson's formula, Schwarz's theorem, reflection principle.

UNIT- III

Power series expansions – The Weierstrass theorem, The Taylor series, The Laurent series.

UNIT- IV

Partial fractions and factorization – Partial fractions, Mittag - Leffer's theorem, Infinite products, Canonical products, The Gamma and Beta functions, Sterling's formula. Entire functions – Jensen's formula, Hadamard's theorem.

Books for Reference:

1. L. V. Ahlfors – Complex Analysis, McGraw-Hill, Kogakusha,1979.

2. J. B. Conway – Functions of one complex variable, Narosa, NewDelhi.

3. R. P. Boas – Invitation to Complex Analysis, The Random House,1987.

4. B. C. Palka – An Introduction to the Complex Function Theory, Springer,1991.

5. S. Ponnusamy – Foundations of Complex Analysis, Narosa,1995.

SOFT CORE

COURSE-IV: ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS

Course Outcome:

- Solve problems in ordinary differential equations, dynamical systems, stability theory, and a number of applications to scientific and engineering problems.

- Demonstrate their ability to write coherent mathematical proofs and scientific arguments needed to communicate the results obtained from differential equation models.

- Demonstrate their understanding of how physical phenomena are modeled by differential equations and dynamical systems.

- Implement solution methods using appropriate technology.

- Investigate the qualitative behavior of solutions of systems of differential equations and interpret in the context of an underlying model.

Course Content

UNIT- I

Linear Second Order Equations - Initial value problem, Existence and Uniqueness by Picard's Theorem, Wronskian, separation and comparison theorems, Poincare phase plane, variation of parameters.

UNIT- II

Power series solutions - Solution near ordinary and regular singular point. Convergence of the formal power series, applications to Legendre, Bessel, Hermite, Laguerre and hypergeometric differential equations with their properties.

UNIT- III

Partial differential equations - Cauchy problems and characteristics, Classification of Second order PDE's, reduction to canonical forms, derivation of the equations of mathematical physics and their solutions by separation of variables.

UNIT-IV

Boundary value problems - Transforming Boundary value problem of PDE and ODE, Sturm - Liouvile system, eigen values and eigen functions, simple properties, expansion in eigen functions, Parseval's identity, Green's function method.

Books for Reference:

1. E. A. Coddington and N. Levinson – Theory of Ordinary Differential equations, Tata McGraw-Hill, NewDelhi.

2. R. Courant and D. Hilbert – Methods of Mathermatical Physics, Vol. I. & II, Tata McGraw-Hill, New Delhi,1975.

3. G. F. Simmons – Differential Equations with applications and Historical Notes, Tata McGraw-Hill, NewDelhi, 1991.

4.   I. N. Sneddon – Theory of Partial differential equations, McGraw-Hill, International StudentEdition.

5.   S. G. Deo and V. Raghavendra – Ordinary Differential Equations and Stability Theory, Tata McGraw-Hill, NewDelhi.

COURSE-V: REPRESENTATION THEORY OF FINITE GROUPS

Course Outcome:

- Utilize Classical groups to solve different related problems.
- Identify and analyze different types of algebraic structures such as G – invariant spaces and Group algebra,
- Design, analyze and implement various representation of symmetric groups for solving different types of problems.
- Identify the challenging problems in modern algebra and find their appropriate solutions.

Course Content

UNIT -I

Classical Groups: General linear group, Orthogonal group, Symplectic group, Unitary group.

UNIT- II

Group representation, Conjugate representation, G-invariant spaces – irreducible representations – Schur's lemma.

UNIT- III

The Group Algebra – Maschke's theorem – characters. Orthogonality relations for characters – Number of irreducible representations.

UNIT- IV

Permutation representations – Regular representation. Representations of Symmetric groups. Representation of Finite abelian groups – Dihedral groups.

Books for Reference:

1. Alperin, J. L.; Bell R. B., **"Groups** and Representations", Graduate Texts in Mathematics, 162, Springer-Verlag, New York, 1995.

2. Curtis C. W.; Reiner I., **"Representation** theory of finite groups and associative algebras", Pure and Applied Mathematics, Vol. XI Interscience Publishers, a division of John Wiley & Sons, New York-London 1962.

3. Dummit D. S.; Foote R. M. **"Abstract** algebra", Third edition, John Wiley & Sons, Inc., Hoboken, NJ, 2004.

4. Fulton; Harris, **"Representation** theory: A first course" Graduate Texts in Mathematics, 129, Readingsin Mathematics, Springer-Verlag, New York, 1991.

5. James, Gordon; Liebeck, Martin, **"Representations** and characters of groups", Second edition, Cambridge University Press, New York, 2001.

6. Musili C. S., **"Representations** of finite groups" Texts and Readings in Mathematics, Hindustan Book Agency, Delhi, 1993.

OPEN ELECTIVE

COURSE-VI: DISCRETE MATHEMATICS

Course Outcome:

- Describe and apply formal definitions.

- Conduct and explain basic formal proofs.

- Use models of computation and specification.

- Use combinatorial reasoning.

- Understand the definitions namely, cut vertex, bridge, blocks and Automorphism group of a graph.

- Study the properties of trees and connectivity.

Course Content

UNIT- I

Mathematical Logic: Connection – Normal Forms – Theory of Inferences – Predicate Calculus.

UNIT- II

Set Theory: Operations on Sets – Basic Set Identities – Relations and Orderings.

UNIT- III

Recursion: Functions – Recursive Functions – Partial Recursive Functions.

UNIT- IV

Graph Theory: Basic Concepts of Graph Theory- Paths – Connectedness –
Matrix Representation of Graphs – Trees – List structures and Graphs

Books for Reference:

1. C. L. Liu – Elements of Discrete Mathematics, McGraw-Hill,1986.

2. Kenneth H. Rosen – Discrete Mathematics and its Applications, McGraw-Hill, 2002.

3. F. Harary – Graph Theory, Addition Wesley Reading Mass, 1969.

4. N. Deo – Graph Theory With Applications to Engineering and Computer Science,
    Prentice Hall of India,1987.

5. K. R. Parthasarathy – Basic Graph Theory, Tata McGraw-Hill, New Delhi,1994.

6. G. Chartand and L. Lesniak – Graphs and Diagraphs, wadsworth and Brooks, 2nd Ed.,

7. Clark and D. A. Holton – A First Look at Graph Theory, Alliedpublishers.

8. D. B. West – Introduction to Graph Theory, Pearson Education Inc.,2001, 2ndEd.,

9. J. A. Bondy and U. S. R. Murthy – Graph Theory with applications, Elsevier,1976

THIRD SEMESTER

HARD CORE

COURSE-I: ELEMENTS OF FUNCTIONAL ANALYSIS

Course Outcome:

After completing this course, the student will be able to:

- Learn to recognize the fundamental properties of normed spaces and of
  the transformations between them.

- Acquaint with the statement of the Hahn-Banach theorem and its corollaries.

- Understand the notions of dot product and Hilbert space.

- Understand the statements and proofs of important theorems and be able to explain the key steps in proofs, sometimes with variation.

Course Content

UNIT -I

Metric completion. Banach's contraction mapping theorem and applications, Baire' category theorem, Ascoli - Arzela theorem.

UNIT- II

Linear spaces and linear operators, Norm of a bounded operator, The Hahn – Banach extension theorem, Stone - Weirstrass theorem.

UNIT -III

Open mapping and Closed Graph theorems. The Banach - Steinhaus Principle of Uniform Boundedness.

UNIT -IV

Hilbert spaces- The orthogonal projection, Nearly orthogonal elements, Riesz's lemma, Riesz's representation theorem.

Books for Reference:

1. G. F. Simmons – Introduction to Topology and Modern Analysis, Tata McGraw-Hill, NewDelhi.

2. A. E. Taylor – Introduction to Functional Analysis, Wiley, New York,1958.

3. A. Page and A. L. Brown – Elements of FunctionalAnalysis.
4. George Bachman and Lawrence Narici – Functional Analysis, Dover Publications, Inc., Mineola, NewYork.

5. J. B. Conway – A Course in Functional Analysis, GTM, Vol. 96., Springer,1985.

COURSE-II: TOPOLOGY I (TOPOLOGY - FCBCS)

Course Outcome:

After completing this course, the student will be able to:

- Distinguish among open and closed sets on different topological spaces.
- Know the two fundamental topologies: discrete and indiscrete topologies
- Identify precisely when a collection of subsets of a given set equipped with a topology forms a topological space
- Understand when two topological spaces are homeomorphic.
- Differentiate different concepts of distance between two sets, connectedness, denseness, compactness and separation axiom

Course Content

UNIT -I

Set theoretic preliminaries.Topological spaces and continuous maps - topological spaces, basis for a topology, the order topology, the product topology on X xX, the subspace topology.

UNIT- II

Closed sets and limit points, continuous functions, the product topology, the metric topology, the quotient topology.

UNIT- III

Connectedness - connected spaces, connected sets on the real line, path connectedness.

UNIT- IV

Compactness - compact spaces, compact sets on the line, limit point compactness, local compactness.

Books for Reference:

1. J. R. Munkres – A First Course in Topology, Prentice Hall India, 2000, 2ndEd.,

2. G. F. Simmons – Introduction to Topology and Modern Analysis, McGraw-Hill, Kogakusha,1968.

3. S. Willard – General Topology, Addison Wesley, New York,1968.

4. J. Dugundji – Topology, Allyn and Bacon, Boston,1966.

5. J. L. Kelley – General Topology, Van Nostrand and Reinhold Co., New York,1955.

SOFT CORE

COURSE-III: THEORY OF NUMBERS

Course Outcome:

- Understand all the basic concepts of Number Theory.

- Learn different Arithmetical Functions and apply to the distribution of lattice points visible from the origin.

- Study properties of Approximation to Irrational numbers.

- Analyze the complicated structure of continued fractions and their number theoretic properties.

Course Content

UNIT- I

Prime numbers, The Fundamental theorem of Arithmetic, The series of Reciprocals of primes, The Euclidean Algorithm. Fermat and Mersenne numbers. Farey series, Farey dissection of the continuum, Irrational numbers-Irrationality of mth **root of N, e andπ.**

UNIT- II

Arithmetical Functions – The Mobius function, The Euler' function and Sigma function, The Dirichlet product of Arithmetical functions, Multiplicative functions. Averages of Arithmetical functions – Euler summation formula, Some elementary asymptotic formulas, The average orders of $d(n)$, $\sigma(n)$, $\varphi(n)$, $\mu(n)$. An application to the distribution of lattice points visible from theorigin.

UNIT- III

Approximation Irrational numbers, Hurwitz's Theorem, Representation of a number by two or four squares, Definition $g(k)$ and $G(k)$, Proof of $g(4)<50$, Perfect numbers. The series of Fibonacci and Lucas.

UNIT- IV

Continued fractions - Finite continued fractions, Convergent of a continued fraction, Continued fractions with positive quotients. Simple continued fractions, The representation of an irreducible rational fraction by a simple continued fraction. The continued fraction algorithm and Euclid's algorithm. The difference between the fraction and its convergents, Infinite simple continued fractions, the representation of an irrational number by an infinite continued fraction, Equivalent numbers and periodic continued fractions, some special quadratic surds.

Books for Reference:

1. G. H. Hardy and E. M. Wright – An Introduction to Theory of Numbers, Oxford University Press, 1979, 5thEd.,

2. I. Niven, H. S. Zuckerman and H. L. Montgomery – An Introduction to the Theory of Numbers, New York,John Wiley and Sons, Inc., 2004, 5thEd.,

3. Bruce C. Berndt – Ramanujan's Note Books Volume-1 to 5,Springer.

4. G. E. Andrews – Number Theory, Dover Books,1995.

5. T. M. Apostol – Introduction to Analytic Number Theory, Narosa Publishing House, NewDelhi.

COURSE-IV: GRAPH THEORY

Course Outcome:

- Understand the definitions namely, cut vertex, bridge, block and block graph.
- Study the properties of trees and connectivity.
- Study Spanning trees and Partitions.
- Discuss and understand the importance of the concepts of Coverings and Independence.

Course Content

UNIT- I

Types of Graphs, Walks and connectedness, Degrees, External graphs, Intersection graphs and operations on graphs

UNIT- II

Cut points, Bridges and Blocks, Block graphs and Cut point graphs.

UNIT- III

Characterization of trees, centers and centroids, Spanning trees and Partitions.

UNIT -IV

Connectivity and line-connectivity, Menger's theorem, Coverings and Independence, Critical points and lines.

Books for Reference:

1. F. Harary – Graph Theory, Addition Wesley Reading Mass, 1969.

2. N. Deo – Graph Theory With Applications to Engineering and Computer Science, Prentice Hall of India,1987.

3. K. R. Parthasarathy – Basic Graph Theory, Tata McGraw-Hill, New Delhi,1994.

4. G. Chartand and L. Lesniak – Graphs and Diagraphs, Qwadsworth and Brooks, 2ndEd.,

5. Clark and D. A. Holton – A First Look at Graph Theory, Alliedpublishers.

6. D. B. West – Introduction to Graph Theory, Pearson Education Inc., 2001, 2ndEd.,

7. J. A. Bondy and U. S. R. Murthy – Graph Theory with applications, Elsevier,1976.

COURSE-V: THEORY OF MODULAR FORMS

Course Outcome:

- Get good exposure to highly complicated unimodular functions.▯
- Learn Mobius transformation and its application to Picard's theorem.▯
- Work on Dedekind's functional equation and its rich structure.▯
- Prove deep theorems on Congruences for the coefficients of the modular function j.▯

Course Content

Unit I.

Elliptic Functions Introduction, Doubly periodic functions, Fundamental pairs of periods, Elliptic functions, Construction of elliptic functions, The Weierstrass $\wp$function, The Laurent expansion of$\wp$near the origin, Differential equation satisfied by $\wp$, The Eisenstein series and the invariants $g2$ and $g3$,The numbers $e1$, $e2$, $e3$ The discriminant Δ, Klein's modular function $J(\tau)$, Invariance of J under unimodular transformations, The Fourier expansions of $g2$ (τ) and $g3$ (τ), The Fourier expansions of $\Delta(r)$ and $J(\tau)$.

Unit II.

The Modular group and modular functions Mobius transformations, The modular group Γ, Fundamental regions, Modular functions, Special values of J, Modular functions as rational functions of J, Mapping properties of J, Application to the inversion problem for Eisenstein series, Application to Picard's theorem.

Unit III.

The Dedekind eta function Introduction, Siegel's proof of Theorem 3.1, Infinite product representation for $\Delta(\tau)$, The general functional equation for $\eta(\tau)$ transformation formula , Deduction of Dedekind's functional equation from Iseki's Formula, Properties of Dedekind sums, The reciprocity law for Dedekind sums, Congruence properties of Dedekind sums , The Eisenstein series $G2$ (τ).

Unit IV.

Congruences for the coefficients of the modular function j Introduction, The subgroup $\Gamma0$ (q), Fundamental region of $\Gamma0$ (p), Functions automorphic under the subgroup $\Gamma0$ (p), Construction of functions belonging to $\Gamma0$ (p), The behavior of fp under the generators ofΓ, The function $\varphi(\tau) = \Delta(q\tau)/\Delta(\tau)$, The univalent function $\phi(\tau)$, Invariance of $\phi(\tau)$ under transformations of $\Gamma0$ (q), The function jp expressed as a polynomial in ϕ.

Books for Reference:

1. Tom M. Apostol, Modular Functions and Dirichlet Series in Number Theory, Springer - Verlag, 1976..

2. Gunning, R. C. Lectures on Modular Forms, Annals of Mathematics Studies, No. 48. Princeton Univ. Press, Princeton, New Jersey, 1962. MR 24 # A2664.

COURSE-VI: COMMUTATIVE ALGEBRA

Course Outcome:

- Comprehend the important concepts and results of Rings and Ideals.

- Prove deep theorems on different kinds of Ideals.

- Apply standard theorems on Homeomorphisms to algebraic problems.

- Learn advanced concepts such as Artinian and Noetherian modules.

Course Content

UNIT- I

Rings and ideals - Rings and ring homomorphisms, Ideals, Quotient rings, zero-divisors, nilpotent elements, units, prime ideals and maximal ideals.

UNIT- II

The prime spectrum of a ring, the nil radical and Jacobson radical, operation on ideals, extension and contraction.

UNIT- III

Modules - Modules and modules homomorphisms, submodules and quotient modules,

Direct sums, Free modules Finitely generated modules, Nakayama Lemma, Simple modules,

Exact sequences of modules.

UNIT- IV

Modules with chain conditions - Artinian and Noetherian modules, modules of finite length, Artnian rings, Noetherian rings, Hilbert basis theorem.

Books for Reference:

1. M. F. Atiyah and I. G. Macdonald – Introduction to Commutative Algebra, Addison-Wesley.

2. C. Musili – Introduction to Rings and Modules, Narosa Publishing House.

3. Miles Reid – Under-graduate Commutative Algebra, Cambridge University Press.

4. N. S. Gopalakrishnan, Commutative Algebra, Oxonian Press.

COURSE-VII: ALGEBRAIC NUMBER THEORY

Course Outcome:

- Apply Number theory to unique factorization domain.
- Prove Ramanujan - Nagell theorem.
- Learn Dedekind domains in the context of algebraic number theory.
- Differentiate class group and class number for better understanding.

Course Content

UNIT- I

Number theoretical applications of unique factorization - Algebraic integers.

UNIT- II

Quadratic fields, Certain Euclidean rings of algebraic integers, Some Diophantine equations, Ramanujan - Nagell theorem.

UNIT- III

Factorization of Ideals - Dedekind domains, Fractional ideals, Invertible ideals, Prime factorization of ideals.

UNIT -IV

Class group and Class number, Finiteness of the Class group, Class number computations.

Books for Reference:

1. Karlheinz Spindler – Abstract Algebra with Applications, Vol. II, Rings and Fields, Marcel Dekker, Inc.

2. I. N. Stewart and David Tall – Algebraic Number Theory, Chapman and Hall.

3. Jody Esmonde and M. Ram Murthy – Problems in Algebraic Number Theory, Springer Verlag.

4. I. S. Luthar and I. B. S. Passi – Algebra Vol. II: Rings, Narosa Publishing House.

COURSE-VIII: GALOIS THEORY

Course Outcome:

- Show familiarity with the concepts of ring and field, and their main algebraic properties;

- Correctly use the terminology and underlying concepts of Galois theory in a problem-solving context☐

- Reproduce the proofs of its main theorems and apply the key ideas in similar arguments;☐

- Calculate Galois groups in simple cases and to apply the group-theoretic information to deduce results about fields and polynomials☐

Course Content

UNIT- I

Algebraically closed fields and algebraic closures, The existence of an algebraic closure, The basic isomorphisms of algebraic field theory, Automorphisms and fixed fields, The Frobenius automorphism, The isomorphism extension theorem.

UNIT- II

The index of a field extension, Splitting fields, Separable extensions, Perfect fields, Normal extensions.

UNIT- III

Galois theory - the main theorem of Galois theory, Galois groups over finite fields, Symmetric functions, Cyclotomic extensions, Constructible numbers.

UNIT- IV

The impossibility of certain geometrical constructions, constructible polygons, Subnormal and normal series, the Jordan- Holder theorem, Radical extensions and solution of equation by radicals, The insolvability of the quintic.

Books for Reference:

1.   J. B. Fraleigh – A First Course in Abstract Algebra, Narosa Publishing House.

2.   Ian Steward – Galois Theory, Chapman and Hall.

3.   Joseph Rotman – Galois Theory, Universitext Springer, 1998.

4.   Michael Artin – Algebra, Prentice-Hall of India, New Delhi.

5.   Joseph A. Gallian – Contemporary Abstract Algebra, Narosa Publishing House, 4th Ed.,

6.   D. S. Dummit and R. M. Foote – Abstract Algebra, John Wiley and Sons, 1999.

7.   I. N. Herstein – Topics in Algebra, Vikas Publishing House, New Delhi.

8.   N. S. Gopalakrishnan – University Algebra, New Age International, 2nd Ed.,

COURSE-IX: DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS

Course Outcome:

- Analyze real world scenarios to recognize when ordinary differential equations (ODEs) or systems of ODEs are appropriate, formulate problems about the scenarios, creatively model these scenarios (using technology, if appropriate) in order to solve the problems using multiple approaches, judge if the results are reasonable, and then interpret and clearly communicate the results.☐

- Recognize ODEs and system of ODEs concepts that are encountered in the real world, understand and be able to communicate the underlying mathematics involved to help another person gain insight into the situation.☐

- Work with ODEs and systems of ODEs in various situations and use correct mathematical terminology, notation, and symbolic processes in order to engage in work, study, and conversation on topics involving ODEs and systems of ODEs with colleagues in the field of mathematics, science or engineering.☐

Course Content

UNIT- I

Recap of Elementary Functions of Calculus - Properties of limits, derivatives and integrals

of elementary functions of Calculus, Polynomials, Rational functions, exponential and

logarithmic functions, trigonometric and inverse trigonometric functions, hyperbolic and

inverse hyperbolic functions.

UNIT- II

Special Functions of Mathematical Physics - Bessel functions, Legendre polynomials,

Tchebyshev polynomials, Hermite polynomials and Laguerre polynomials. Power series

solutions of Second Order Linear Differential Equations. Their Mathematical properties.

UNIT- III

Applications of First Order Ordinary Differential Equations - Simple problems of dynamics – falling bodies and other motion problems, Simple problems of Chemical reactions and mixing, Simple problems of growth and decay.

UNIT- IV

Applications of Second Order Ordinary Differential Equations - Undamped simple harmonic motion, damped vibrations, Forced vibrations, Problems on simple electric circuits – Laplace transforms.

Books for Reference:

1. G. F. Simmons, Differential Equations with Applications and Historical Notes, Tata McGraw-Hill, New Delhi,1991.

2. E. D. Rainville and P. Bedient– Elementary course on Ordinary Differential Equations, Macmillan, New York,1972.

3. R. Courant and D. Hilbert, Methods of Mathematical Physics, Vol. I, Tata McGraw- Hill, New Delhi,1975.

FOURTH SEMESTER

HARD CORE

COURSE-I: MEASURE AND INTEGRATION

Course Outcome:

After completing this course, the student will be able to:

- Describe and apply the notion of measurable functions and sets and use Lebesgue monotone and dominated convergence theorems and Fatous Lemma.

- Describe the construction of and apply the Lebesgue integral.

- Describe the construction of product measures and use Fubini's theorem.

- Describe the notion of absolute continuity and singularities of measures and apply Lebesgue decomposition and the Radon-Nikodym theorem.

Course Content

UNIT- I

Lebesgue measure - outer measure, measurable sets and Lebesgue measure, a nonmeasurable set, measurable functions.

UNIT- II

The Lebesgue integral – the Lebesgue Integral of a bounded function over as set of

finite measure, the integral of a non-negative function, the general Lebesgue integral.

UNIT- III

Differentiation and integration - Differentiation of monotonic functions, functions of bounded variation, differentiation of an integral, absolute continuity.

UNIT- IV

Measure and integration - Measure spaces, Measurable functions, integration, Signed measures, the Radon - Nikodym theorem, Measure and outer measure, outer measure and measurability, the extension theorem, product measures.

Books for Reference:

1. H. L. Royden – Real Analysis, Prentice Hall, 3rdEd.,

2. G. de Barra – Measure Theory and Integration, Wiley EasternLimited.

3. Inder K. Rana – An Introduction to Measure and Integration, Narosa,1997.

COURSE-II: TOPOLOGY II (Advanced Topology - FCBCS)

Course Outcome:

- Distinguish among countability and separation axioms on different topological spaces.

- Understand Urysohn's metrization theorem.

- Learn Tychonoff's theorem on the product of compact spaces.

- Apply Fundamental groups in algebraic topology.

Course Content

UNIT-I

Countability and Separation axioms-the countability axioms, these parathion axioms, normality of a compact Hausdorff space.

UNIT-II

Urysohn's lemma, Tietze's extension theorem, Urysohn's metrization theorem, Partitions of unity.

UNIT- III

Tychonoff's theorem on the product of compact spaces. Local finiteness, Para compactness, Normality of a paracompact space.

UNIT- IV

The Fundamental group and the Fundamental group of a circle, The Fundamental group of the punctured plane, Essential and Inessential Maps, The Fundamental Theorem of Algebra.

Books for Reference:

1. James R. Munkres - A First Course in Topology , Prentice Hall India, 2000, 2nd Ed.,

2. G. F. Simmons – Introduction to Topology and Modern Analysis, McGraw-Hill, Kogakusha,1968.

3. S. Willard – General Topology, Addison Wesley, New York,1968.

4. J. Dugundji – Topology, Allyn and Bacon, Boston,1966.

5. J. L. Kelley – General Topology, Van Nostrand and Reinhold Co., New York,1955.

SOFT CORE

COURSE-III: DIFFERENTIAL GEOMETRY

Course Outcome:

- Explain the concepts and language of differential geometry and its role in modern mathematics.

- Analyze and solve complex problems using appropriate techniques from differential geometry.
- Apply problem-solving with differential geometry to diverse situations in physics, engineering or other mathematical contexts.
- Apply differential geometry techniques to specific research problems in mathematics or other fields.

Course Content

Unit I

Plane curves and Space curves – Frenet-Serret Formulae. Global properties of curves –

Simple closed curves, The isoperimetric inequality, The Four Vertex theorem. Surfaces in three

dimensions – Smooth surfaces, Tangents, Normals and Orientability, Quadric surfaces.

UNIT-II

The First Fundamental form – The lengths of curves on surfaces, Isometries of

surfaces, Conformal mappings of surfaces, Surface area, Equiareal Maps and a

theorem of Archimedes.

UNIT- III

Curvature of surfaces – The Second Fundamental form, The Curvature of curves on a surface, Normal and Principal Curvatures.

UNIT- IV

Gaussian Curvature and The Gauss' Map – The Gaussian and The mean Curvatures, The Pseudo sphere, Flat surfaces, Surfaces of Constant Mean Curvature, Gaussian Curvature of Compact surfaces, The Gauss' Map.

Books for Reference:

1. A. Pressley – Elementary Differential Geometry, Under-graduate Mathematics Series,Springer.

2. T. J. Willmore – An Introduction to Differential Geometry, Oxford UniversityPress.

3. D. Somasundaram – Differential Geometry: A First Course, Narosa,2005.

COURSE-IV: ADVANCED GRAPH THEORY

Course Outcome:

- Distinguish between Eulerian graphs, Hamiltonian graphs and apply to the problem of traversability.
- Understand Plane, Planar graphs and Euler's beautiful formula.
- Apply theory of matrices to Graph theory.
- Analyze concepts of graphs through Group theory.

Course Content

UNIT- I

Traversability - Eulerian graphs, Hamiltonian graphs. Line Graphs - Some properties of line graphs, Characterization of line graphs, Special line graphs, Line graphs and traversability.

UNIT- II

Factorization. Planarity - Plane and planar graphs, Euler's formula, Characterizations of planar graphs, Nonplanar graphs, Outerplanar graphs.

UNIT- III

Colorability - the chromatic number, Five color theorem. Matrices – The adjacency matrix, The incidence matrix, The cycle matrix.

UNIT- IV

Groups – The automorphism group of a graph, Operation on Permutation groups,The group of a composite graph, Graphs with a given group, Symmetric graphs, Highly symmetricgraphs. Domination Theory - Domination numbers, Some elementaryproperties.

Books for Reference:

1. F. Harary – Graph Theory, Addition Wesley Reading Mass, 1969.

2. N. Deo – Graph Theory With Applications to Engineering and Computer Science, Prentice Hall of India,1987.

3. K. R. Parthasarathy – Basic Graph Theory, Tata McGraw-Hill, New Delhi,1994.

4. G. Chartand and L. Lesniak – Graphs and Diagraphs, Qwadsworth and Brooks, 2ndEd.,

5. Clark and D. A. Holton – A First Look at Graph Theory, Alliedpublishers.

6. D. B. West – Introduction to Graph Theory, Pearson Education Inc., 2001, 2ndEd.,

7. J. A. Bondy and U. S. R. Murthy – Graph Theory with applications, Elsevier,1976.

COURSE-V: THEORY OF PARTITIONS

Course Outcome:

- Learn Generating functions for various partitions.

- Prove combinatorically Euler's beautiful identity on pentagonal numbers.

- Understand congruence properties of Rogers - Ramanujan Identities.

- Apply elementary series – product identities to various types of partitions.

Course Content

UNIT -I

Partitions - partitions of numbers, the generating function of p(n), other generating functions, two theorems of Euler, Jacobi's triple product identity and its applications.

UNIT- II

$_1\psi_1$ - summation formula and its applications, combinatorial proofs of Euler's identity, Euler's pentagonal number theorem, Franklin's combinatorial proof.

UNIT- III

Congruence properties of partition function, the Rogers - Ramanujan Identities.

UNIT- IV

Elementary series - product identities, Euler's, Gauss', Heine's, Jacobi's identities. Restricted Partitions – Gaussian, Frobinius partitions.

Books for Reference:

1. G. H. Hardy and E. M. Wright – An Introduction to Theory of Numbers, Oxford University Press, 1979, 5th Ed.,

2. I. Niven, H. S. Zuckerman and H. L. Montgomery – An Introduction to the Theory of Numbers, New York, John Wiley and Sons, Inc., 2004, 5th Ed.,

3. Bruce C. Berndt – Ramanujan's Note Books Volumes-1 to 5.

4. G. E. Andrews – The Theory of Partitions, Addison Wesley, 1976.

5. A. K. Agarwal, Padmavathamma, M. V. Subbarao – Partition Theory, Atma Ram & Sons, Chandigarh, 2005.

COURSE-VI: HYPERGEOMETRIC FUNCTIONS AND q – SERIES

Course Outcome:

After completing this course, the student will be able to:

- Learn the Gauss Function.
- Prove combinatorically Basic Hypergeometric Functions.
- Understand Theta Function identities found in Chapter 16 of Ramanujan's second notebook.
- Analyze complicated Rogers-Ramanujan continued fractions and related theta-function identities.

Course Content

UNIT-I: The Gauss Function

Historical introduction, The Gauss series and its convergence, The Gauss equation,

The connection with Riemann's equation, Kummer's twenty-four solutions,

Contiguous functions and recurrence relations, Special cases of the Gauss function,

Some integral representations, The Gauss summation theorem, Another special

summation theorem, Analytic continuation formulae.

UNIT-II: Basic Hypergeometric Functions

Convergence of Heine series-Some simple results, Jackson's Theorem, Basic analogue of Saalschutz's Theorem, Application of Bailey's transformation to basic series, Some numerical evaluation of infinite products, Basic bilateral series, Ramanujan's $_1\psi_1$ - summation formula.

UNIT-III: Theta Functions

Ramanujan's general theta-function and its particular cases, Theta-function

identities of Ramanujan found in his Chapter 16 of his second notebook, Quintuple

product identity and its applications.

UNIT-IV: q-Continued fractions

Ramanujan's cubic continued fractions, Rogers-Ramanujan continued fractions and

related theta-function identities.

Books for Reference:

1. L. J. Slater, Generalized Hypergeometric Functions, Cambridge University Press, London, 1966.

2. H. Exton, q-Hypergeometric Functions and Applications, Ellis Horwood Series in Mathematics and its Application, Chichester, 1983.

3. B. C. Berndt, Ramanujan's Notebooks, Part III, Springer-Verlag, New York, 1991.

4. G. Gasper and M. Rahman, On Basic Hypergeometric Series, Second Ed., Encycl. Math. Applics., Vol 35, Cambridge University Press, Cambridge, 2004.

5. B. C. Berndt, Number Theory in the Spirit of Ramanujan, American Mathematical Society, Providence, RI 2006.

COURSE-VII: ADVANCED FUNCTIONAL ANALYSIS

Course Outcome:

- Explain the fundamental concepts of functional analysis and their role in modern mathematics and applied contexts.
- Demonstrate accurate and efficient use of functional analysis techniques.
- Demonstrate capacity for mathematical reasoning through analyzing, proving and explaining concepts from functional analysis.
- Apply problem-solving using functional analysis techniques applied to diverse situations in physics, engineering and other mathematical contexts

Course Content

UNIT-I

Bounded linear operators on Hilbert spaces, the adjoint of an operator, self adjoint operators, positive operators, properties of normal and unitary operators. One to one correspondence between projections on a Banach space and pairs of closed linear subspaces of the space, properties of orthogonal projections on Hilbert spaces.

UNIT- II

Spectral resolution of an operator on a finite dimensional Hilbert space H and the spectral theorem of a normal operator on H.

UNIT- III

The structure of commutative Banach algebras - properties of the Gelfand mapping, the maximal ideal space, multiplicative functional and the maximal ideal.

UNIT- IV

Applications of spectral radius formula. Involutions in Banach algebras, the Gelfand - Neumark theorem.

Books for Reference:

1. G. F. Simmons – Introduction to Topology and Modern Analysis, Tata McGraw- Hill, New Delhi.

2. A. E. Taylor – Introduction to Functional Analysis, Wiley, New York, 1958.

3. A. Page and A. L. Brown – Elements of Functional Analysis.

4. George Bachman and Lawrence Narici – Functional Analysis, Dover Publications, Inc., Mineola, New York.

5. J. B. Conway – A course in Functional Analysis, GTM, Vol. 96., Springer, 1985.

OPEN ELECTIVE

COURSE-VIII: ALGORITHMS AND COMPUTATIONS

Course Outcome:

- Select a computation problem to work on.
- Analyze a problem and its complexity.
- Analyze several algorithmic solutions to the problem.
- Use one or more appropriate models of computation in the analysis.
- Select and/or generate input data for the algorithmic solutions.
- Analyze empirical results to get a deeper understanding of the algorithmic solutions and/or the problem.

Course Content

UNIT- I

Introduction to Computers, Flowcharts, Algorithms and their features, Languages, Types of language and translators. Numerical Algorithms - Solving a simultaneous system of linear equations using iterative and direct methods.

UNIT- II

Interpolation algorithms - equal, unequal intervals, central difference and inverse interpolation. Numerical differentiation and integration and their errors calculations.

UNIT-III

Graph theoretical algorithms - Connectivity, finding shortest path between two vertices, enumeration of all paths, construction of minimum spanning tree, cutest, cut vertex, coding and decoding.

UNIT- IV

Computation - Algorithms complexities, strategies, Divide and conquer, greedy technique, Introduction to NP hard problems.

Books for Reference:

1. Conte and D'bear – Numerical Algorithms, McGraw-Hill, 1985.

2. N. Deo – Graph Theory With Applications to Engineering and Computer Science, Prentice Hall of India, 1987.

3. E. V. Krishnamurthy – Introductory Theory of Computer Science, Prentice Hall of India, 1980.

4. Horowitz and Sahni – Fundamentals of Computer Algorithms, Addison Wesley, 1987.

5. V. Rajaraman – Computer Oriented Numerical Methods, Prentice Hall of India, 1980.

6. G. Shankar Rao – Numerical Analysis, Prentice Hall of India, 1985.